2025

2nd Semester Examination

MATHEMATICS (Honours)

Paper : C 4-T

[Differential Equations and Vector Calculus]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***ten*** questions : 2×10=20

   (a) If S is defined by the rectangle $|x| \le a, |y| \le b$, show that the function $f(x,y) = x\sin y + y\cos x$, satisfy the Lipschitz condition. Find the Lipschitz constant.

   (b) Verify that origin is regular singular point of the equation $2x^2y'' + xy' - (x+1)y = 0$.

   (c) Find the linear differential equation with real constant coefficients that is satisfied by the function $y = 9 - 3x + \frac{1}{6}e^{5x}$.

P.T.O.

(d) Find the indicial roots of the differe

$x^2y''+xy'+(x^2-4)y=0$ near $x=0$

(e) If e^{2x} and $x\,e^{2x}$ are particular solutions

order homogeneous differential wit

coefficients, then find the differential equ

(f) Find the recurrence relation for the

series solution of the differential e

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2+2)y=0.$$

(g) Find the volume of the parallelepiped whose

are represented by

$$\vec{a}=2\hat{i}-4\hat{j}+5\hat{k},\ \vec{b}=\hat{i}-\hat{j}+\hat{k},\ \vec{c}=3\hat{i}-5\hat{j}+$$

(h) Given that

$$\vec{r}=4a\sin^3\theta\,\hat{i}+4a\cos^3\theta\,\hat{j}+3b\cos 2\theta\,\hat{k},$$

find $\left|\frac{d\vec{r}}{d\theta}\times\frac{d^2\vec{r}}{d\theta^2}\right|$.

(i) Evaluate $\int \vec{a}\cdot\left(\vec{r}\times\frac{d^2\vec{r}}{dt^2}\right)dt$.

(j) Show that one solution of $\frac{xdx}{y^2z}=\frac{ydy}{xz}=\frac{dz}{y^2}$ represents a hyperbola.

(k) Define the critical point and isolated point in a plane autonomous system.

(l) Discuss the singularity of the differential equation

$$x^2(x-2)^2\frac{d^2y}{dx^2}+2(x-2)\frac{dy}{dx}+(x+1)y=0.$$

(m) Determine the nature of the critical point (0,0) plane autonomous system $\dot{x}=5x-2y,\ \dot{y}=-2x+5y$.

(n) With the use of undetermined coefficients, find the particular integral of $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=x^2e^x$.

(o) Factorise $\left[xD^2+(x-1)D-1\right]y=x^2$ and reduce it to a first order differential equation.

5×4=20

2. Answer any *four* questions :

(a) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2}+a^2y=\sec ax.$$

(b) Solve the system of simultaneous equations

$$2\frac{d^2y}{dx^2}-\frac{dz}{dx}-4y=2x;\ 2\frac{dy}{dx}+4\frac{dz}{dx}-3z=0.$$

(c) Prove that

$$\left[\vec{a}\times\vec{p},\ \vec{b}\times\vec{q},\ \vec{c}\times\vec{r}\right]+\left[\vec{a}\times\vec{q},\ \vec{b}\times\vec{r},\ \vec{c}\times\vec{p}\right]+\left[\vec{a}\times\vec{r},\ \vec{b}\times\vec{p},\ \vec{c}\times\vec{q}\right]=0$$

P.T.O.

(d) Prove that $\nabla^2(r^n\vec{r})=n(n+3)r^{n-2}\vec{r}$.

(e) Show $\vec{F}=\left(2xy+z^3\right)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a
vative force field. Find the scalar potential.

(f) Given that $y=e^{2x}$ is a solution of

$$(2x+1)\frac{d^2y}{dx^2}-4(x+1)\frac{dy}{dx}+4y=0,$$ fin

linearly independent solution by reducing the
Write the general solution.

3. Answer any *two* questions : 10×2

(a) (i) Find the series solution of the equation

$$\left(x^2-1\right)y''+4xy'+2y=0 \text{ near } x=0.$$

(ii) If $\vec{r}(t)=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$, prove that

$$\int_1^2 \vec{r}\times\frac{d^2\vec{r}}{dt^2}dt=-14\hat{i}+75\hat{j}-15\hat{k}.$$

7+3

(b) (i) Find the nature of the critical point (0, 0) of the linear autonomous system.

$$\frac{dx}{dt}=x+7y,\ \frac{dy}{dt}=3x+5y$$

Discuss the stability of the critical point. Find the general solution. Draw a diagram in the phase plane.

(ii) Show that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\vec{b}\vec{c}\right]^2$.

8+2

(c) (i) Solve the system : $\dfrac{dy}{dt}+2\dfrac{dx}{dt}+x+5y=4t$

and $\dfrac{dy}{dt}+\dfrac{dx}{dt}+2x+2y=2$.

(ii) In what direction from the point (1,1, –1) is the directional derivative of
$\varphi=x^2-2y^2+4z^2$ a maximum?
What is the magnitude of this maximum directional derivative?

5+5

(d) (i) Solve the differential equation

$$\frac{d^2y}{dx^2}+(x-1)^2\frac{dy}{dx}-4(x-1)y=0,$$ in series

about the point $x = 1$.

(ii) Calculate $\int_{(0,0,0)}^{(1,1,1)}\vec{F}\cdot d\vec{r}$,

$$\vec{F}=(y^2+z^2)\hat{i}+(x^2+z^2)\hat{j}+(x^2+y^2)\hat{k}$$

along the path $t\hat{i}+t^2\hat{j}+t^3\hat{k}$.

6+4